

Mathematical Investigation: Point of No Return

(Collaborative Task: Forms groups of 2-3 people.)

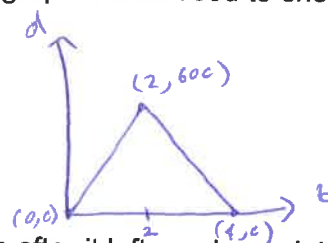
In this investigation the relationship between the speed and distance of travel of a plane will be examined, combined with any wind effects which it experiences. The concept of “the point of no return” will be encountered, i.e. the point after which the plane cannot return to its starting point, but must continue on its journey. After first analysing this problem with specific values and graphing these results this “point of no return”.



Part 1

Imagine that you are the pilot of the light aircraft, which is capable of cruising at a steady speed of 300 kph in still air. There is enough fuel on board to last for four hours. Consider firstly the situation on a perfectly calm day, i.e. no wind. A graphical approach will be used to solve the problem. A graph will be used to show how distance from the airfield varies with time, i.e. plot Distance versus Time.

- Mark a point on the graph to represent the plane's initial position.
 - Draw a line to represent the outward speed of 300 kph.
 - What feature of the graph gives the speed directly? *gradient*
 - Assuming that the plane arrives back at the airfield exactly four hours after it left, mark a point on the graph to represent its final position.
 - Draw a line to represent the return speed of 300 kph.
 - Find the maximum distance that the pilot can fly from the airfield, and still be sure that there is enough fuel to make a safe return journey – this is “the point of no return”, and state what time it takes to reach this point.
- ~~1200 km~~ 600 km 2 hours (Total distance is 1200 km)*
- Mark this point of no return clearly on your graph.
-



Mark this point of no return clearly on your graph.

- Construct two linear equations that when solved simultaneously would give you the 'point of no return'. Check your results with the results above.

Going out
 $d = 300 \text{ t}$

$$d - 600 = -300(t - 2)$$

$$\begin{aligned} d &= -100t + 600 \\ d &= -300t + 1200 \end{aligned}$$

$$300t = -300t + 1200$$

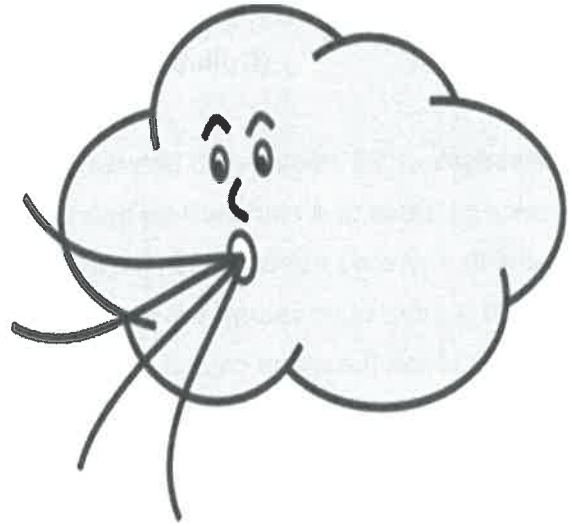
$$600t = 1200$$

$t = 2$

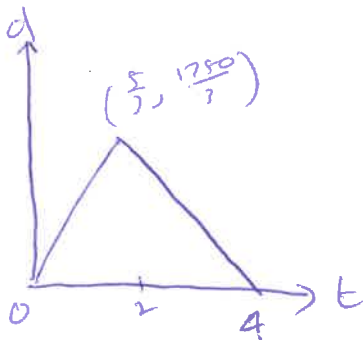
$$d = 50^\circ$$

Part 2

Consider now the situation when it is no longer calm, i.e. there is a wind blowing.



- Consider now the case when the plane takes off from the airfield and, on the outward journey, is helped along by a 50 kph wind (i.e. a "tail wind") which increases cruising speed relative to the ground to 350 kph. Suddenly the pilot realises that on the return journey the plane will be flying into the wind and will therefore slow down to 250 kph. Again, there is only enough fuel on board to last for four hours.
 - Use the graph to show an outward speed of 350 kph.
 - Use the graph to show a return speed of 250 kph.
 - Mark on the graph the new "point of no return" and state its distance from the airfield.
 - What time will it take to reach this point?
- Consider another wind speed, this time a 50 kph head wind on the outward journey. Repeat the calculations above, and mark the new "point of no return" on the graph.
- Repeat these 2 calculations with 2 other wind speeds of your own choice, showing calculations and plotting these points on the graph also.
- If these points were joined to form a smooth curve, what shape do you think the curve would have?



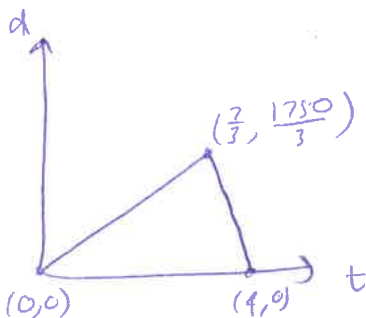
$$d = 350t \quad d - 0 = -250(t - 4)$$

$$d = -250t + 1000$$

$$350t = -250t + 1000$$

$$600t = 1000$$

$$t = \frac{5}{3} \quad \Bigg| \quad d = \frac{1750}{3}$$



$$d = 250t \quad d - 0 = -150(t - 4)$$

$$d = -150t + 600$$

$$600t = 600$$

$$t = \frac{2}{3} \quad \Bigg| \quad d = \frac{1750}{3}$$

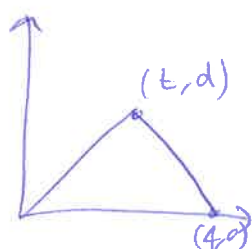
Part 3

Suppose the wind speed is w kph, the "point of no return" is d km from the airfield and the time at which the pilot should turn around is t hours. You can assume that there is only enough fuel on board to last for four hours, and the plane is capable of cruising at a steady speed of 300kph in still air.

- Write down two expressions for the *outward* speed of the aircraft, one involving w and one involving d and t .
- Write down two expressions for the *homeward* speed of the aircraft, one involving w and one involving d and t .
- Express d in terms of only t by eliminating w from the two resulting equations.
- Does this explain the pattern made by your "points of no return"? Why/why not?

Extension

- What wind speeds – head or tail – would prevent the plane from completing a journey?
- Consider the plane being capable of cruising at a steady speed of a kph. Repeat part 3. Express d in terms of t and a .



outward speed

$$300 + w = \frac{d}{t}$$

$$300 + w = \frac{d}{t}$$

$$w = \frac{d}{t} - 300$$

homeward speed

$$300 - w = \frac{d}{4-t}$$

$$300 - w = \frac{d}{4-t}$$

$$300 - w = \frac{d}{4-t}$$

Speed must be positive

$$300 - \left(\frac{d}{t} - 300\right) = \frac{d}{4-t}$$

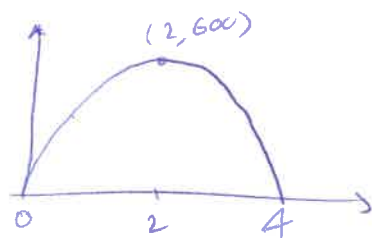
$$600 = \frac{d}{4-t} + \frac{d}{t}$$

$$600 = \frac{dt + d(4-t)}{t(4-t)}$$

$$600t(4-t) = 4d$$

$$d = 150t(4-t)$$

parabola



• Wind speed can't exceed 300 kph

$$-300 \leq w < 300$$

$$\begin{cases} 300 - w > 0 \\ \& \\ 300 + w > 0 \end{cases}$$

$$a + w = \frac{d}{t} \quad \& \quad a - w = \frac{d}{4-t}$$

$$2a = \frac{d}{t} + \frac{d}{4-t}$$

$$2a = \frac{d(4-t) + dt}{t(4-t)}$$

$$2at(4-t) = 4d$$

$$d = \frac{1}{2}at(4-t)$$

Point of no return

$$\left(\frac{2(a-w)}{a}, \frac{2}{a}(a^2 - w^2) \right)$$

Total distance travelled is $\frac{4}{a}(a^2 - w^2)$

